Study of the order of the phase transition in pure U(1) gauge theory with Villain action

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We address the question of the order of the deconfinement phase transition of four dimensional U(1) lattice gauge theory. Simulations of the $\mathbb{Z}$-gauge theory dual to the Villain action on toroidal lattices up to lattice sizes of 28^4 give results consistent with both, a vanishing and a nonvanishing discontinuity in the thermodynamic limit. A decision on the order of the phase transition requires still larger lattice sizes.

1. INTRODUCTION

U(1) lattice gauge theory with Villain action

$$\exp(-S_{\mathrm{V}}) = \prod_{\mathrm{P}} \sum_{m_{\mathrm{P}} \in \mathbb{Z}} \exp\left\{-\frac{\beta}{2}(\theta_{\mathrm{P}} - 2\pi m_{\mathrm{P}})^2\right\} \quad (1)$$

is, on the infinite lattice, equivalent [1] to the $\mathbb{Z}$ gauge theory

$$S_{\mathrm{Z}} = \kappa \sum_{\mathrm{P}} n_{\mathrm{P}}^2 \qquad \left(\kappa = \frac{1}{2\beta}\right), \quad (2)$$

a gauge theory with an integer valued gauge field on the links. In particular the order of the phase transition and universal quantities are the same. We choose the $\mathbb{Z}$ gauge theory for computational advantages and use a toroidal lattice with periodic boundary conditions. We expect[1] the Villain action to have properties similar to those of the extended action

$$S_{\mathrm{E}} = -\sum_{P}(\beta\cos(\theta_P) + \gamma\cos(2\theta_P)) \quad (3)$$

with $\gamma \approx -0.22$.

The order of the phase transition in this parameter region is controversial. Scaling of gauge-balls [2] suggests second order. Finite size scaling of the specific heat is consistent with first order [3,4] (torus and surface of 5D hypercube)

[1]This is suggested by a Fourier transform of S_{V}.

as well as with second order [5,6] (homogenous sphere). The volume dependence of the latent heat on finite lattices is consistent with first order as well as with second order.

For a more detailed discussion of our results we refer to [7].

2. SIMULATION

A Metropolis algorithm is used for the update. The statistics is summarized in Tab. 1. We perform Ferrenberg Swendsen multihistogram reweighting [8] to the maximum of the specific heat. Jackknife errors are calculated for all quantities. The flip autocorrelation time τ_{flip}, which corresponds to the mean life time of a pure phase configuration, is estimated from the time evolution of the energy density. τ_{flip} is a more adequate measure for the number of statistically independent measurements than τ_{int}. τ_{flip} is in our case about a factor 3-5 greater than τ_{int}.

3. SPECIFIC HEAT

Finite size scaling theory for first order transitions (see e.g. [9]) predicts the maximum of the specific heat to diverge $\propto L^D$ which implies $\nu = 1/4$. The effective exponent $\nu_{\mathrm{eff}}(L)$, which is calculated from the specific heat at two neighbouring L, is shown in Fig. 1. We compare with ν_{eff} obtained in [4] at $\gamma = -0.2$ and find agreement almost within error bars.
Clearly there is a tendency for $\nu_{\mathrm{eff}}(L)$ to decrease over the L range considered, but a stabilization

L^4	κ	$\kappa_{\mathrm{pc}}(C_{\mathrm{V}})$	Msw.	τ_{int}	τ_{flip}
6^4	0.77955	0.780350(18)	29	101	-
8^4	0.77716	0.778332(24)	10.5	233	-
	0.77822		14.1	228	-
10^4	0.77713	0.777556(21)	4.3	502	-
	0.77745		1.035	479	-
12^4	0.77706	0.777183(12)	2.05	887	-
	0.77715		11.5	935	-
14^4	0.77699	0.776976(07)	9.9	1697	9000
16^4	0.77689	0.776882(11)	7.8	2852	12500
18^4	0.77681	0.776816(07)	4.42	4384	20000
	0.77685		1.525	5156	19000
20^4	0.77677	0.776786(06)	2.9	8639	27000
	0.77680		1.05	7084	31000
22^4	0.77676	0.776760(06)	2.55	12903	43000
	0.77677		2.862	11383	47000
24^4	0.77674	0.776742(05)	2.0	16360	40000
	0.77675		2.1	17807	50000
26^4	0.776745	0.776758(10)	1.42	35151	110000
28^4	0.77672	0.776726(07)	0.95	28701	120000

Table 1

Lattice sizes, κ values and positions of the specific heat maxima. The statistics is given in 10^6 sweeps. Also included are the integrated autocorrelation time and an estimate of the flip duration in sweeps.

on a certain value is not observed.

4. PROBABILITY DISTRIBUTION

The probability distribution $p_L(e)$, where e is the action divided by the number of plaquettes, has for $L \geq 10$ two peaks with a minimum in between them. It is not well described by the sum of two Gaussians. The minimum becomes more pronounced with increasing volume, but the distance between the peaks decreases. The histograms are reweighted to the maximum of the specific heat. For a first order transition this is asymptotically equivalent to equal weight of the coexisting phases. Then the positions in e of the maxima, the minimum and where the histogram is half of its respective maximum value on the outer flanks of the distribution function are calculated.

A first order transition would obviously require the width of $p_L(e)$ to extrapolate to a non-zero value. It is consistent with the data that the histogram width shrinks to zero approximately like $1/L$ for $L \longrightarrow \infty$ (Fig. 2). This behaviour would imply a second order phase transition in spite of the observation of hysteresis and a double peaked
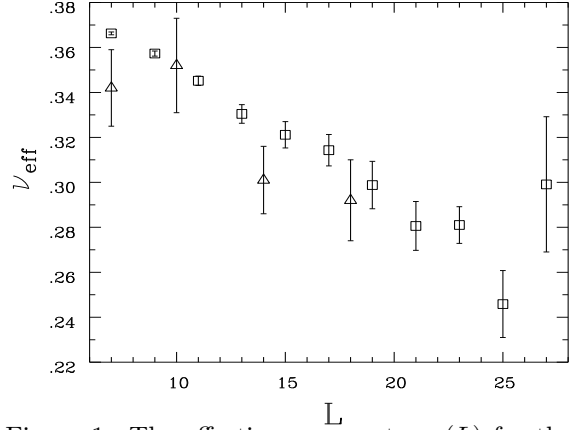

Figure 1. The effective exponent $\nu_{\mathrm{eff}}(L)$ for the Villain action (1) (squares) and the extended action (3) (triangles) at $\gamma = -0.2$. The values for the extended action are taken from Fig. 15 in [4].

histogram on finite lattices.

The position especially of the lower maximum (confinement phase) varies irregularly. Presumably the discrete nature of the considered action contributes to this effect.

The L extrapolation of the upper energy branch's data (Coulomb phase) is less sensitive to the extrapolation ansatz than the lower energy branch's data. This is related to the fact that the energy fluctuations within the confinement branch are much larger than in the Coulomb phase. In fact they have the magnitude of the possible gap itself.

Following [10], we use

$$F(L) = \frac{1}{2L^{d-1}} \log \left(\frac{P_L^{\mathrm{max},1} P_L^{\mathrm{max},2}}{(P_L^{\mathrm{min}})^2} \right) \qquad (4)$$

as a finite volume estimator for the interface tension. $F(L)$ slowly decreases with the lattice size. Again a stabilization to a constant value is not observed. It may shrink to zero for $L \to \infty$. A power law fit $F(L) \propto L^{-z}$ results in $z = 0.3(1)$.

Also the ratio of the peak heights is not stable in the considered L range.

5. CONCLUSIONS

We have done a high statistics simulation of the $\mathbb{Z}$ gauge theory using large lattices. Nevertheless, it is not possible to give definite answers to the question of the order of the phase transition from
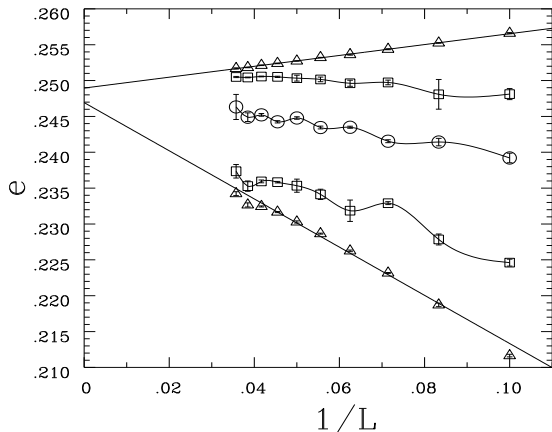
Figure 2. The positions of the maxima (squares), of the minimum (circles) of the energy density probability distribution, and the position on the flanks, where the probability distribution has half the value of the respective maximum (triangles).

our simulations. We find that the typical indications of a first order transition — hysteresis and histograms with double peaks — persist up to $L = 28$. On the other hand, the width of the energy density histogram might shrink to zero, which would imply second order. The L dependent exponent $\nu_{\text{eff}}(L)$ decreases steadily but does not stabilize in the considered L range. The limit $\lim_{L \to \infty} \nu_{\text{eff}}(L)$ requires larger lattices. The finite volume estimator for the interface tension slowly decreases with L and might vanish in the thermodynamic limit. In addition, the flattening of the probability distribution in the metastable phase region typical for first order phase transitions has not been observed in our simulations. Therefore a stabilization of the finite volume interface tension estimator is not expected. The ratio of the peak heights at the maximum of C_V does not stabilize in the considered L range.

The simulations have been done on toroidal lattices as opposed to spherical lattices with homogenously distributed curvature [5,6], where full consistency with second order finite size scaling behaviour has been seen. Also, we want to point out that a mass determination on toroidal lattices away from the transition point — where finite size effects are under control —, as has been done in [2], is feasible in spite of the complications immediately at the transition point. Those results favor second order. Our data for the effective $\nu_{\text{eff}}(L)$ indicate that if the phase transition should actually turn out to be first order, the next larger lattice sizes may exhibit a stabilization of $\nu_{\text{eff}}(L)$ at the value 1/4. Such lattices could, in principle, be simulated on massively parallel architectures.

The transition might well be of first order, but even if that was a given fact larger lattices would be necessary for a better understanding of the peculiar finite size effects. In that case the similarity to second order would indicate a critical point not far away in the coupling space. In any case it will be interesting to pursue the investigation of pure U(1) gauge theory on the lattice further.

6. ACKNOWLEDGEMENTS

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